

**WEEKLY TEST TYJ TEST - 13 RAJPUR ROAD**  
**SOLUTION Date 13-07-2019**

**[PHYSICS]**

1.

2.

3. In return journey the angle made by the projectile with the horizontal is  $-\theta$ . Hence,

$$\vec{\Delta p} = [\hat{i}p \cos \theta + \hat{j}p \sin \theta]$$

$$-[\hat{i}p \cos(-\theta) + \hat{j}p \sin(-\theta)] = \hat{j}2p \sin \theta$$

or  $|\vec{\Delta P}| = 2p \sin \theta$

4. 
$$\frac{h_2}{h_1} = \frac{u^2 \sin^2 \theta_2}{2g} \times \frac{2g}{u^2 \sin^2 \theta_1}$$

$$= \frac{\sin^2 \theta_2}{\sin^2 \theta_1} = \frac{\sin^2 \pi/6}{\sin^2 \pi/3} = \frac{1}{3}$$

$\therefore h_2 = h_1/3$

5. Time taken by bullet to reach the target =  $\frac{\text{distance}}{\text{velocity}} = \frac{\text{distance}}{u \cos \theta}$

As  $\theta$  is very small,  $\cos \theta = 1$

$$\text{Time} = \frac{\text{distance}}{u} = \frac{400}{400} = 1 \text{ sec}$$

Vertical deflection of bullet

$$= \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (1)^2 = 5 \text{ metre}$$

6.  $R_{\max.} = \frac{u^2}{g} = 1000 \text{ m}$  (R is maximum where  $\theta = 45^\circ$ )

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2}{2g} \times \sin^2 45^\circ = \frac{u^2}{4g}$$

$$= \frac{1000}{4} = 250 \text{ m}$$

7. Kinetic energy is minimum at the highest point and highest point is attained after covering distance equal to  $0.5 R$

8. When the horizontal range is maximum, the maximum height attained is  $R/4$ . Hence, co-ordinates of the point =  $(400, 100)$ .

9. R is same for both  $\theta$  and  $(90^\circ - \theta)$ ,

If angle w.r.t. vertical is  $40^\circ$ , then w.r.t. horizontal direction it will be  $90^\circ - 40^\circ = 50^\circ$ .

10. 
$$\frac{R}{T^2} = \frac{u^2 \sin 2\theta \times g^2}{g \cdot 4u^2 \sin^2 \theta} = \frac{g}{2} \cot \theta$$

i.e.,  $gT^2 = 2R \tan \theta$

If T is doubled, then R becomes 4 times



11. The shooter has to direct slightly upward as the path followed by a projectile is a parabola trajectory.

$$12. H_{\max.} = \frac{u^2 \sin^2 \theta}{2g}$$

$$T = \frac{2u \sin \theta}{g}$$

$$\frac{H_{\max.}}{T^2} = \frac{u^2 \sin \theta}{2g} \times \frac{g^2}{4u^2 \sin^2 \theta}$$

$$= \frac{g}{8} = \frac{10}{8} = \frac{5}{4}$$

$$13. H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{or } 80 = \frac{u^2 \sin^2 \theta}{2 \times 10}$$

$$\text{or } u^2 \sin^2 \theta = 1600$$

$$\text{or } u \sin \theta = 40 \text{ ms}^{-1}.$$

$$\text{Horizontal velocity} = u \cos \theta = at$$

$$= 3 \times 30 = 90 \text{ ms}^{-1}$$

$$\frac{u \sin \theta}{u \cos \theta} = \frac{40}{90}$$

$$\text{or } \tan \theta = \frac{4}{9} \quad \text{or} \quad \theta = \tan^{-1} \left( \frac{4}{9} \right)$$

$$14. \theta_1 = 30^\circ, \quad \theta_2 = 60^\circ$$

$$H = \frac{u^2 \sin^2 \theta}{2g}, \quad \text{i.e.,} \quad H \propto \sin^2 \theta$$

$$\therefore \frac{H_1}{H_2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_2} = \frac{\sin^2 30^\circ}{\sin^2 60^\circ}$$

$$= \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3}$$

15. Component of velocity  $\perp$  to plane remains the same (in opposite direction),  
i.e.,  $u \sin \theta = 20 \sin 30^\circ = 10 \text{ m/s}$ .

### [MATHEMATICS]

31. (c) Given, diameter of circular wire = 10cm, therefore length of wire =  $10\pi$ .

$$\text{Hence required angle} = \frac{\text{arc}}{\text{radius}} = \frac{10\pi}{50} = \frac{\pi}{5} \text{ radian.}$$

32.

(d) Given expression is

$$\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 85^\circ + \sin^2 90^\circ.$$

$$\text{We know that } \sin 90^\circ = 1 \text{ or } \sin^2 90^\circ = 1.$$

Similarly,  $\sin 45^\circ = \frac{1}{\sqrt{2}}$  or  $\sin^2 45^\circ = \frac{1}{2}$  and the angles are in A.P. of 18 terms. We also know that

$$\sin^2 85^\circ = [\sin(90^\circ - 5^\circ)]^2 = \cos^2 5^\circ.$$

Therefore from the complementary rule, we find  $\sin^2 5^\circ + \sin^2 85^\circ = \sin^2 5^\circ + \cos^2 5^\circ = 1$ .

Therefore,

$$\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 85^\circ + \sin^2 90^\circ$$

$$= (1+1+1+1+1+1+1) + 1 + \frac{1}{2} = 9 \frac{1}{2}.$$



33. (b) We have  $x + \frac{1}{x} = 2 \cos \theta$ ,

$$\begin{aligned} \text{Now } x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3x \frac{1}{x} \left(x + \frac{1}{x}\right) \\ &= (2 \cos \theta)^3 - 3(2 \cos \theta) = 8 \cos^3 \theta - 6 \cos \theta \\ &= 2(4 \cos^3 \theta - 3 \cos \theta) = 2 \cos 3\theta. \end{aligned}$$

**Trick :** Put  $x = 1 \Rightarrow \theta = 0^\circ$ .

$$\text{Then } x^3 + \frac{1}{x^3} = 2 = 2 \cos 3\theta.$$

34. (a)  $\sin x + \operatorname{cosec} x = 2 \Rightarrow (\sin x - 1)^2 = 0 \Rightarrow \sin x = 1$   
 $\therefore \sin^n x + \operatorname{cosec}^n x = 1 + 1 = 2$ .

35. (b) Since  $\cos^6 \alpha + \sin^6 \alpha + K \sin^2 2\alpha = 1$  using formula  $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$  and on solving, we get the required result i.e.  $K = \frac{3}{4}$ .

36. (c)  $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{1}{2} \sin 20^\circ \sin 60^\circ (2 \sin 40^\circ \sin 80^\circ)$

$$\begin{aligned} &= \frac{1}{2} \sin 20^\circ \sin 60^\circ (\cos 40^\circ - \cos 120^\circ) \\ &= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \sin 20^\circ \left(1 - 2 \sin^2 20^\circ + \frac{1}{2}\right) \\ &= \frac{\sqrt{3}}{4} \sin 20^\circ \left(\frac{3}{2} - 2 \sin^2 20^\circ\right) \\ &= \frac{\sqrt{3}}{8} (3 \sin 20^\circ - 4 \sin^3 20^\circ) = \frac{\sqrt{3}}{8} \sin 60^\circ = \frac{\sqrt{3}}{8} \cdot \frac{\sqrt{3}}{2} = \frac{3}{16}. \end{aligned}$$

37. (b)  $x = \sin 130^\circ \cos 80^\circ$ ,  $y = \sin 80^\circ \cos 130^\circ$   
 $\Rightarrow x = \cos 40^\circ \cos 80^\circ$ ,  $y = -\sin 80^\circ \sin 40^\circ$   
 So,  $x > 0$  and  $y < 0$  and  $xy < 0$   
 Now,  $z = 1 + xy \Rightarrow 0 < z < 1$ .

38. (a, c) We have  $\sin \alpha = 1/\sqrt{5} \Rightarrow \cos \alpha = 2/\sqrt{5}$   
 and  $\sin \beta = 3/5 \Rightarrow \cos \beta = 4/5$   
 $\sin(\beta - \alpha) = \sin \beta \cos \alpha - \sin \alpha \cos \beta$   
 $= \frac{3}{5} \cdot \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}} \cdot \frac{4}{5} = \frac{2}{5\sqrt{5}} = 0.1789$   
 Now  $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = 0.7071 = \sin \frac{3\pi}{4}$   
 Since  $0 < 0.1789 < 0.7071$   
 $\therefore \sin 0 < \sin(\beta - \alpha) < \sin \frac{\pi}{4} \Rightarrow 0 < (\beta - \alpha) < \frac{\pi}{4}$   
 Also,  $\sin \pi < \sin(\beta - \alpha) < \sin \frac{3\pi}{4}$   
 $\therefore (\beta - \alpha) \in [0, \pi/4]$  and  $[3\pi/4, \pi]$ .

39. (b) Let  $f(x) = \sin \theta + \cos \theta = \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right)$   
 But  $-1 \leq \sin\left(\theta + \frac{\pi}{4}\right) \leq 1 \Rightarrow -\sqrt{2} \leq \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right) \leq \sqrt{2}$ .  
 Hence the maximum value of  $(\sin \theta + \cos \theta)$   
 i.e., of  $\sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right) = \sqrt{2}$ .  
 $\therefore \sin\left(\theta + \frac{\pi}{4}\right) = 1 \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \sin \frac{\pi}{2}$   
 $\Rightarrow \theta + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} = 45^\circ$ .
40. (b)  $5 - 5 \sin^2 \theta + 7 \sin^2 \theta = 6 \Rightarrow 2 \sin^2 \theta = 1$   
 $\Rightarrow \sin^2 \theta = \frac{1}{2} = \sin^2\left(\frac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$ .
41. (b)  $\tan \theta = \frac{-1}{\sqrt{3}} = \tan\left(\pi - \frac{\pi}{6}\right)$ ,  $\sin \theta = \frac{1}{2} = \sin\left(\pi - \frac{\pi}{6}\right)$  and  $\cos \theta = \frac{-\sqrt{3}}{2} = \cos\left(\pi - \frac{\pi}{6}\right)$   
 Hence principal value is  $\theta = \frac{5\pi}{6}$ .
42. (a)  $2 \sin 3x \cos x - 2 \sin 3x = 0$ ,  $\therefore \sin 3x = 0$ ,  $\cos x = 1$   
 $\Rightarrow 3x = n\pi$  or  $x = \frac{n\pi}{3}$  and  $x = 2n\pi$   
 The second value  $x = 2n\pi$  is included in the value given by  $x = \frac{n\pi}{3}$ .
43. (c)  $(5 + 4 \cos \theta)(2 \cos \theta + 1) = 0$   
 $\cos \theta = -5/4$ , which is not possible.  
 $\therefore 2 \cos \theta + 1 = 0$  or  $\cos \theta = -1/2$   
 $\Rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ . Solution set is  $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\} \in [0, 2\pi]$ .
44. (a)  $\tan 2\theta = \cot \theta \Rightarrow \tan 2\theta = \tan\left(\frac{\pi}{2} - \theta\right)$   
 $\Rightarrow 2\theta = n\pi + \frac{\pi}{2} - \theta \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{6}$ .
45. (d)  $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = 3 \Rightarrow \frac{1 - (1 - 2 \sin^2 \theta)}{1 + (2 \cos^2 \theta - 1)} = 3$